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The Supernova–GRB connection

Color superconducting quark matter in compact stars

A. Drago[1], A. Lavagno[2], and G. Pagliara[1]

[1] Dipartimento di Fisica, Università di Ferrara and INFN, Sezione di Ferrara, 44100 Ferrara, Italy
[2] Dipartimento di Fisica, Politecnico di Torino and INFN, Sezione di Torino, 10129 Torino, Italy

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Abstract. We study the effects of color superconductivity on the structure and formation of compact stars. We show that it is possible to satisfy most of recent observational boundaries on masses and radii if a diquark condensate forms in a hybrid or a quark star. Moreover, we find that a huge amount of energy, of the order of 10^{53} erg, can be released in the conversion from a (metastable) hadronic star into a (stable) hybrid or quark star, if the presence of a color superconducting phase is taken into account. Accordingly to the scenario proposed in Astrophys.J.586(2003)1250, the energy released in this conversion can power a Gamma Ray Burst. Possible experimental evidences, indicating a range of time delay between a Supernova explosion and a subsequent Gamma Ray Burst, are here discussed and interpreted.

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1 Introduction

The new accumulating data from X-ray satellites provide important information on the structure and formation of compact stellar objects. Concerning the structure, the new data fix rather stringent constraints on the mass and the radius of a compact star. These data are at first sight difficult to interpret in a unique and self-consistent theoretical scenario, since some of the observations are indicating rather small radii and other observations are indicating large values for the mass of the star.

Concerning the formation scenario, crucial information are provided by the very recent observations of Gamma-Ray Bursts (GRB), indicating the possibility that some of the GRBs are associated with a previous Supernova (SN) explosion, with a delay between the first and the second explosion of the order of days or years [1,2]. A delay of order one week is also compatible with the results of the analysis of GRB030329 [3]. These observations could be explained associating the second explosion with the conversion of a (metastable) hadronic star (HS) into a more stable stellar object made at least in part of deconfined quark matter (QM). In the scenario proposed in [4] the HS can be metastable due to the presence of a non-vanishing surface tension at the interface separating hadronic matter (HM) from QM. The nucleation time (i.e. the time to form a critical-size drop of quark matter) can be extremely long if the mass of the star is small. Via mass accretion the nucleation time can be dramatically reduced and the star is finally converted into the stable configuration. The newly formed HyS or QS cools down emitting neutrinos and the subsequent neutrino-antineutrino annihilation can power a GRB.

In recent years, many theoretical works have investigated the possible formation of a diquark condensate in quark matter, at densities reachable in the core of a compact star [5,6,7]. The formation of this condensate can deeply modify the structure of the star [8,9,10].

We show that it is possible to satisfy the existing boundaries on mass and radius of a compact stellar object if a diquark condensate forms in a Hybrid Star (HyS) or a Quark Star (QS). Moreover, the formation of diquark condensate can significantly increase the energy released in the conversion from a purely HS into a more stable star containing deconfined QM.

2 Equation of state of beta-stable matter

To describe the high density EOS of matter we adopt standard models in the various density ranges. Concerning the hadronic phase we use the relativistic nonlinear Glendenning-Moszkowski model (GM1-GM3) [11]. At very low density we have used the Negele-Vautherin [12] and the Baym-Pethick-Sutherland [13] EOS. For the quark matter phase we adopt a MIT-bag like model in which the formation of a diquark condensate is taken into account in a simple and effective way. To connect the two phases of our EOS, we impose Gibbs equilibrium conditions.

It is widely accepted that the Color-Flavor Locking phase (CFL) is the real ground state of QCD at asymp-

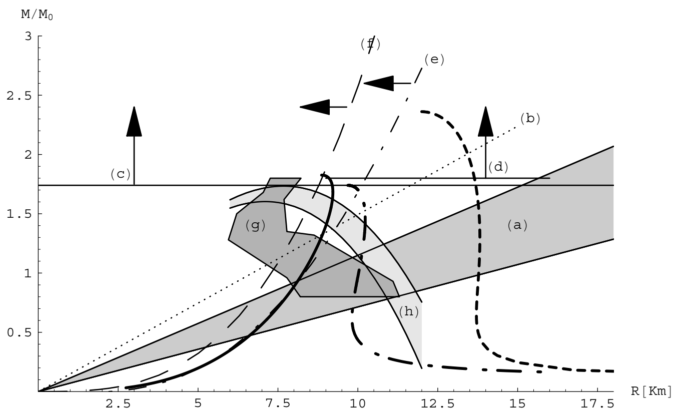

Fig. 1. Mass-radius plane with observational limits and a few representative theoretical curves: *thick solid line* indicates CFL quark stars, *thick dot-dashed line* CFL hybrid stars, *thick-dashed line* hadronic stars (*see text*). The observational limits come from: **a** Sanwal et al. 2002 [15], **b** Cottam et al. 2002 [16], **c** Quaintrell et al. 2003 [17], **d** Heinke et al. 2003 [18], **e,g** Dey et al. 1998 [19], **f** Li et al. 1999 [20], **h** Burwitz et al. 2002 [21]

toticly large densities. We are interested in the bulk properties of a compact star and we adopt the simple scheme proposed in [8,14] where the thermodynamic potential is given by the sum of two contributions. The first term corresponds to a "fictional" state of unpaired quark matter in which all quarks have a common Fermi momentum chosen to minimize the thermodynamic potential. The other term is the binding energy Δ of the diquark condensate expanded up to order $(\Delta/\mu)^2$. In [8] the gap is assumed to be constant, independent on the chemical potential μ. In the present calculation we consider a μ dependent gap resulting from the solution of the gap equation. The resulting EOS in our model reads therefore:

$$P = -\Omega_{CFL}(\mu) - B - \Omega^{electrons}(\mu_e) \quad (1)$$

$$E/V = \Omega_{CFL}(\mu) + \mu\rho + B + \Omega^{electrons}(\mu_e) + \mu_e\rho_e \quad (2)$$

where

$$\Omega_{CFL}(\mu) = \frac{6}{\pi^2}\int_0^\nu k^2(k-\mu)\,\mathrm{d}k + \frac{3}{\pi^2}\int_0^\nu k^2(\sqrt{k^2+m_s^2}-\mu)\,\mathrm{d}k - \frac{3\Delta^2\mu^2}{\pi^2} \quad (3)$$

with

$$\nu = 2\mu - \sqrt{\mu^2 + \frac{m_s^2}{3}}\,, \quad (4)$$

and the quark density ρ is calculated numerically by deriving the thermodynamic potential respect to μ.

3 Masses and radii of compact stellar objects

In Fig. 1 we have collected most of the analysis of data from X-ray satellites, concerning masses and radii of compact stellar objects [15,16,17,18,19,20,21]. Observing Fig. 1, we notice that the constraints coming from a few data sets (labeled "e", "f"[1] "g" and maybe also constraint "h") indicate rather unambiguously the existence of very compact stellar objects, having a radius smaller than $\sim$ 10 km. At the contrary, at least in one case ("a" in the figure), the analysis of the data suggests the existence of stellar objects having radii of the order of 12 km or larger, if their mass is of the order of 1.4 $M_\odot$. In this analysis one has also to take into account that it is difficult from an astrophysical viewpoint to generate compact stellar objects having a mass of the order of one solar mass or smaller. Therefore the most likely interpretation of constraint "a" is that the corresponding stellar object does not belong to the same class of objects which have a radius smaller than $\sim$ 10 km. Concerning constraint "b", its interpretation is less clear, since it can be satisfied both with a very compact star or with a star having a larger radius. The apparent contradiction between the constraints "e", "f", "g" and the constraint "a" can be easily accommodated in our scheme, since it can be the signal of the existence of metastable purely hadronic stars which can collapse into a stable configuration when deconfined quark matter forms inside the star. In the next Section we will discuss the possible relation between this transition and at least some GRBs.

Finally, constraints ("c" and "d"[2]) do not provide stringent limits on the radius of the star, but they put strong constraints on the lower value of its mass. Constraints "c" and "d" are very important, since it is in general not easy to obtain solutions of the Tolman - Oppenheimer - Volkoff equation having both large masses and very small radii. As we will see, the existence of an energy gap associated with the diquark condensate helps in circumventing this difficulty, since the effect of the gap is to increase the maximum mass of QSs or of HySs having a huge content of pure quark matter.

In Fig. 1 we show a few theoretical M-R relations which correspond to the scenario we are proposing. More precisely, we show a thick-dashed line corresponding to HSs (GM1), a thick dot-dashed line corresponding to HySs (GM1, $B^{1/4} = 170$ MeV, Δ_2) and a thick solid line corresponding to QSs (GM1, $B^{1/4} = 170$ MeV, Δ_4). Both the HyS and the QS lines can satisfy essentially all the constrains derived from observations. The shapes of the gaps Δ_i are shown in Fig. 2.

Concerning the constraint "a", it is probably better satisfied by the HS line than by the HyS or QS lines, which would give stars having a mass smaller than $\sim 1.2 M_\odot$. In conclusion, in our scheme most of the compact stars are either HySs or QSs having a mass in the range $1.2 - 1.8 M_\odot$ and a radius $\sim 8.5 - 10$ km. Metastable HS can exist. As we will see in the next section their mass is probably smaller than $\sim 1.3 M_\odot$.

[1] A very recent reanalysis of the data of the pulsar SAX J1808.4-3658, discussed in [20], seems to indicate slightly larger radii, of the order of 9-10 km for a star having a mass of 1.4-1.5 $M_\odot$ [22].

[2] If the observed X-ray emission is due to continuing accretion, then a smaller value for the mass is allowed, $M/M_\odot = 1.4$.

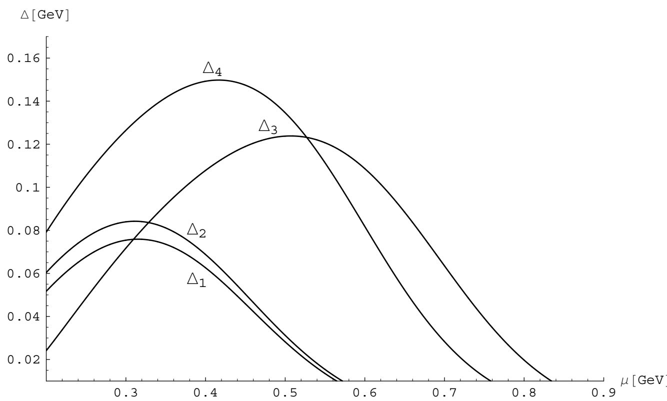

Fig. 2. Gap as function of the chemical potential, for four different parameter sets

4 Nucleation time and energy released

The existence of a possible relation between GRBs and SN explosions has been extensively discussed in the literature. It has not yet been clarified if the two explosions are always simultaneous or if, at least in a few cases, a time delay can exist, with the SN preceding the GRB. A very important information can be obtained from the analysis of the optical afterglow, since in two cases (GRB980425 / SN1998bw, GRB030329 / SN2003dh) the spectrum of a type Ic SN emerged once the spectrum of the GRB afterglow has been subtracted. It is therefore possible, at least in principle, to estimate the date of the SN explosion and to estimate the delay (if any) between the two explosions. Unfortunately, due to the uncertainties on the time dependence of the SN light curves, it is difficult to reduce the error in this estimate to a value smaller than a few days. The analysis of [3] suggests that the SN exploded within a few days of the GRB. More precisely, the difference between the light curves indicates that SN2003dh may have preceded its associated GRB by about 4-7 days. Actually the first indications of a possible delay between SN and GRB came from the analysis of the X-ray spectra of GRB990705 [1] and of GRB011211 [2]. In particular, in the case of GRB990705 the detection (for the first time) of Fe absorption lines indicated the existence of an Fe rich environment crossed by the GRB emission. The Fe abundance was assumed to be generated by a SN explosion preceding the GRB, with a time interval of order years. A similar analysis, performed in [2], suggested a time delay of order days for GRB011211.

The effect of the transition to deconfined QM on explosive processes like SNs and GRBs has been discussed by many authors. In particular, the possibility that the deconfinement transition takes place during the core-collapse of massive stars at the moment of the bounce, has been discussed e.g. in [23,24] and this mechanism can help the SN to explode. However, at the light of results like the ones presented in [25,26], it seems more plausible that the deconfinement takes place only when the proto-neutron star has deleptonized and cooled down to a temperature of a few MeV. In the model we are discussing, proto-neutron stars having a small enough mass can exist as metastable HS if a non-vanishing surface tension is present at the interface between HM and QM. The process of quark deconfinement can be a powerful source for GRBs and it can also explain the delay between a SN explosion and the subsequent GRB observed in a few cases [1,2]. In the model proposed in [4] the central density of a pure HS increases, due to spin down or mass accretion, until its value approaches the deconfinement critical density. At this point a spherical virtual drop of QM can form. The potential energy for fluctuations of the drop radius R has the form [27]:

$$U(R) = \frac{4}{3}\pi R^3 n_q(\mu_q - \mu_h) + 4\pi\sigma R^2 + 8\pi\gamma R \qquad (5)$$

where n_q is the quark baryon density, μ_h and μ_q are the hadronic and quark chemical potentials, all computed at a fixed pressure P, and σ is the surface tension for the interface separating quarks from hadrons. Finally, the term containing γ is the so called curvature energy. For σ we use standard values from 10 to 40 MeV/fm^2 and we assume that it takes into account, in a effective way, also the curvature energy. The value of σ was estimated in [28] to be $\sim$ 10 MeV/fm^2. Values for σ larger than $\sim$ 30 MeV/fm^2 are probably not useful at the light of the result of [29, 30].

To compute the time needed to form a bubble of quarks having a radius larger than the critical one, we use the technique of quantum tunneling nucleation. We can assume that the temperature has no effect in our scheme: for values of $B^{1/4} \sim 160 - 180$ MeV the critical density ρ_1 separating pure HM from mixed phase is larger than $4\rho_0$ for $Z/A \sim 0.3$, i.e. for an isospin fraction typical of a newly formed and hot proto-neutron star [31]. This critical density typically exceeds the central density of hot and not too massive stars. Therefore the mixed phase can form only when the star has deleptonized and its temperature has dropped down to a few MeV [25,26]. When the temperature is so low, only quantum tunneling is a practicable mechanism.

The calculation proceed in the usual way: after the computation (in WKB approximation) of the ground state energy E_0 and of the oscillation frequency ν_0 of the virtual QM drop in the potential well $U(R)$, it is possible to calculate in a relativistic frame the probability of tunneling as [32]:

$$p_0 = \exp[-\frac{A(E_0)}{\hbar}] \qquad (6)$$

where

$$A(E) = 2\int_{R_-}^{R_+} dR\sqrt{[2M(R) + E - U(R)][U(R) - E]}\,. \qquad (7)$$

Here $R_\pm$ are the classical turning points and

$$M(R) = 4\pi\rho_h\left(1 - \frac{n_q}{n_h}\right)^2 R^3 \quad , \qquad (8)$$

ρ_h being the hadronic energy density and n_h, n_q are the baryonic densities at a same and given pressure in the hadronic and quark phase, respectively. The nucleation time is then equal to

$$\tau = (\nu_0 p_0 N_c)^{-1}\,, \tag{9}$$

where N_c is the number of centers of droplet formation in the star, and it is of the order of 10^{48} [32].

Let us recall once again the astrophysical scenario we have in mind. In a few cases a delay of the order of days or years between the SN explosion and the subsequent GRB have been postulated to explain the astrophysical data on the GRBs. In the scheme we are discussing, this delay is due to the formation of a metastable HS having a relatively small mass. The nucleation time, computed using (9), can be extremely long if the mass of the metastable star is small enough. Via mass accretion the nucleation time can be reduced from values of the order of the age of the universe down to a value of the order of days or years. We can therefore determine the critical mass M_{cr} of the metastable HS for which the nucleation time corresponds to a fixed small value (1 year in Table 1).

Table 1. Energy released ΔE (*measured in foe=10*^{51} *erg*) in the conversion to hybrid or quark star (*labeled with a* •), for various sets of model parameters, assuming the hadronic star mean life-time $\tau = 1$ yr (*see text*). M_{cr} is the gravitational mass of the hadronic star at which the transition takes place, for fixed values of the surface tension σ and of the mean life-time τ. BH indicates that the hadronic star collapses to a Black Hole. We indicate with a *dash* (–) situations in which the Gibbs construction does not provide a mechanically stable EOS

Hadronic Model	$B^{1/4}$ [MeV]	σ [MeV/fm^2]	$M_{cr}/M_\odot$	ΔE $\Delta = 0$	ΔE Δ_1	ΔE Δ_2	ΔE Δ_3	ΔE Δ_4
GM3	160	20	0.69	20	65•	69•	76•	148•
GM3	160	30	0.91	32	90•	95•	106•	196•
GM3	160	40	1.00	38	100•	105•	119•	216•
GM3	170	10	1.12	0	34	40	68	162•
GM3	170	20	1.26	4	44	50	86	185•
GM3	170	30	1.39	11	53	60	104	207•
GM3	170	40	1.49	BH	62	68	120	224•
GM3	180	10	1.55	BH	11	13	BH	–
GM3	180	20	1.61	BH	BH	22	BH	–
GM3	180	30	1.67	BH	BH	BH	BH	–
GM1	160	10	0.45	11	41•	44•	47•	96•
GM1	160	20	0.72	28	75•	79•	86•	160•
GM1	160	30	0.96	48	108•	114•	127•	220•
GM1	160	40	1.18	72	142•	148•	166•	276•
GM1	170	10	1.17	18	59	65	96	191•
GM1	170	20	1.33	33	79	85	124	226•
GM1	170	30	1.45	50	96	103	150	254•
GM1	170	40	1.60	BH	122	128	BH	290•
GM1	180	10	1.63	BH	BH	72	BH	–
GM1	180	20	1.72	BH	BH	BH	BH	–
GM1	180	30	1.79	BH	BH	BH	BH	–

In Table 1 we show the value of M_{cr} for various sets of model parameters. In the conversion process from a metastable HS into an HyS or a QS a huge amount of energy ΔE is released. ΔE is the difference between the gravitational mass of the metastable HS and that of the final HyS or QS having the same baryonic mass. We see in the table that the formation of a CFL phase allows to obtain values for ΔE which are one order of magnitude larger than the corresponding ΔE of the unpaired QM case ($\Delta = 0$). Moreover, we can observe that ΔE depends both on magnitude and position of the gap.

In the model we are presenting, the GRB is due to the cooling of the justly formed HyS or QS via neutrino - antineutrino emission. The subsequent neutrino-antineutrino annihilation generates the GRB. In our scenario the duration of the prompt emission of the GRB is therefore regulated by two mechanisms: 1) the time needed for the conversion of the HS into a HyS or QS, once a critical-size droplet is formed and 2) the cooling time of the justly formed HyS or QS. Concerning the time needed for the conversion into QM of at least a fraction of the star, the seminal work by [33] has been reconsidered by [34]. The conclusion of this latter work is that the stellar conversion is a very fast process, having a duration much shorter than 1s. On the other hand, the neutrino trapping time, which provides the cooling time of a compact object, is of the order of a few ten seconds [35], and it gives the typical duration of the GRB in our model.

5 Conclusions

We have studied the effect of color superconductivity on the EOS of quark matter and on the mass-radius relation for hybrid and quark stars. Comparing the theoretical curves with recent analysis of observational data, we find that color superconductivity is a crucial ingredient in order to satisfy all the constraints coming from observations. The most difficult problem posed by the astrophysical data is the indication of the existence of stars which are both very compact (R $\lesssim$ 9–10 km) and rather massive ($M \gtrsim 1.7 M_\odot$). We can satisfy these constraints either with hybrid or quark stars. In particular, concerning hybrid stars, the gap increases the maximum mass of the stable configuration, while keeping the corresponding radius $\lesssim$ 10 km.

The superconducting gap affects also deeply the energy released in the conversion from hadronic star into hybrid or quark star. In the scenario proposed in [4] the transition to deconfined quark matter takes place only when the star has deleptonized and cooled down. This is in agreement with the results of [25,26] and at variance with the scenario proposed in [23,24] where quark matter is produced at the moment of the bounce. This open the possibility to explain recent observations indicating a possible delay between a SN explosion and the subsequent Gamma Ray Burst [1,2,3], since we associate the second explosion with the transition from a metastable hadronic star to a stable star containing deconfined quark matter. The energy released, which will power the Gamma Ray

Burst through neutrino-antineutrino annihilation, is significantly increased by the effect of the superconducting gap and it can reach a value of the order of 10^{53} erg.

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